Name:_____**Key**_____   Date:_____

HW Pre Calculus 11 6.4 Solving Rational Functions

1. Solve each of the following equations. Indicate any extraneous roots if there are any. Show your steps:

a) $\dfrac{4}{x} + \dfrac{3}{x+2} = 5$ LCD: $(x)(x+2)$

$$\dfrac{4}{x}(x)(x+2) + \dfrac{3}{x+2}(x)(x+2) = 5(x)(x+2)$$

$$4x + 8 + 3x = 5x^2 + 10x$$

$$7x + 8 = 5x^2 + 10x$$

$$0 = 5x^2 + 3x - 8$$

$$(5x+8)(x-1)$$

$$\boxed{x = -\dfrac{8}{5}} \quad \boxed{x = 1}$$

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b) $\dfrac{-2}{x+3} - \dfrac{5}{x} = 2$ LCD: $(x)(x+3)$

$$\dfrac{-2}{x+3}(x)(x+3) - \dfrac{5}{x}(x)(x+3) = 2(x)(x+3)$$

$$-2x - 5x - 15 = 2x^2 + 6x$$

$$-7x - 15 = 2x^2 + 6x$$

$$0 = 2x^2 + 13x + 15$$

$$(2x-3)(x+5)$$

$$x = -\dfrac{3}{2} \quad x = -5$$

$\dfrac{3}{x+2} - \dfrac{2}{x-1} = 5$ LCD: $(x+2)(x-1)$

$$\dfrac{3}{x+2}(x+2)(x-1) - \dfrac{2}{x-1}(x+2)(x-1) = 5(x+2)(x-1)$$

$$3x - 3 - 2x - 4 = 5(x^2 + x - 2)$$

$$x - 7 = 5x^2 + 5x - 10$$

$$0 = 5x^2 + 4x - 3$$

$$x = \dfrac{-4 \pm \sqrt{16+60}}{10}$$

) $\dfrac{2}{x+2} + \dfrac{1}{x} = 1$ NPV: $-2, 0$ LCD: $(x+2)(x)$

$$2x + x + 2 = x^2 + 2x$$

$$3x + 2 = x^2 + 2x$$

$$0 = x^2 - x - 2$$

$$(x-2)(x+1)$$

$$x = 2 \quad x = -1$$

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c) $\dfrac{2}{y} = \dfrac{3}{y^2+2}$ NPV: 0 LCD: $(y)(y^2+2)$

$$2y^2 + 4 = 3y$$

$$2y^2 - 3y + 4 = 0$$

$$x = \dfrac{3 \pm \sqrt{9-32}}{4} \quad \leftarrow \text{No REAL solutions}$$

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d) $\dfrac{x}{1} + \dfrac{30}{x+8} = \dfrac{3}{1}$ $x \neq -8$ LCD: $x + 8$

$$x^2 + 8x + 30 = 3x + 24$$

$$x^2 + 5x + 6 = 0$$

$$\begin{array}{cc} x & +3 \\ x & +2 \end{array}$$

$$(x+3)(x-2)$$

$$x = -3 \quad x = -2$$

$$\frac{5}{3x-1} - \frac{9}{6x-1} = 2$$

$$\frac{5(6x-1)(3x-1)}{(3x-1)} - \frac{9}{6x-1}(3x-1)(6x-1) = 2(3x-1)(6x-1)$$

$$30x - 5 - 27x + 9 = 2(18x^2 - 9x + 1)$$

$$3x + 4 = 36x^2 - 18x + 2$$

$$0 = 36x^2 - 21x - 2$$

$$12 \underline{\qquad} -3 = -24$$
$$3 \underline{\qquad} 1 = 3$$

$$0 = (12x + 1)(3x - 2)$$

$$x = -\frac{1}{12}, \frac{2}{3}$$

$$\frac{5}{x+1} + \frac{4}{3} = \frac{x+1}{x-1} \qquad ① LCD\ (x+1)(3)(x-1)$$

$$\frac{5}{(x+1)}(3)(x-1)(x+1) + \frac{4}{3}(3)(x+1)(x-1) = \frac{(x+1)}{(x-1)}(3)(x+1)(x-1)$$

$$15x - 15 + 4(x^2 - 1) = 3(x^2 + 2x + 1)$$

$$4x^2 + 15x - 19 = 3x^2 + 6x + 3$$

$$x^2 + 9x - 22 = 0$$

$$(x+11)(x-2) = 0$$

$$x = -11, x = 2$$

CHK: x=-11
$$\frac{5}{-10} + \frac{4}{3} = \frac{10}{-12} \checkmark$$
$$\frac{-1}{2} + \frac{4}{3} = \frac{5}{6} \checkmark$$

x=2
$$\frac{5}{3} + \frac{4}{3} = \frac{3}{1} \checkmark$$

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e) $$\frac{3x}{x-2} + \frac{x}{x+2} = \frac{2x-1}{x+2} \qquad ① LCD\ (x-2)(x+2)$$

$$\frac{3x}{x-2} = \frac{2x-1}{x+2} - \frac{x}{x+2}$$

$$\frac{3x}{x-2} = \frac{x-1}{x+2}$$

$$3x(x+2) = (x-2)(x-1)$$

$$3x^2 + 6x = x^2 - 3x + 2$$

$$2x^2 + 9x - 2 = 0$$

$$x = \frac{-9 \pm \sqrt{81 + 4(2)(2)}}{4}$$

$$\boxed{x = \frac{-9 \pm \sqrt{97}}{2}}$$

$$\frac{81}{16}$$
$$\frac{97}{}$$

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f) $$\frac{2x-3}{x+2} - \frac{x+2}{x-1} = \frac{3x}{x-1} \qquad ① LCD\ (x+2)(x-1)$$

$$\frac{2x-3}{x+2} = \frac{3x}{x-1} + \frac{x+2}{x-1}$$

$$\frac{2x-3}{x+2} = \frac{4x+2}{x-1}$$

$$(2x-3)(x-1) = (4x+2)(x+2)$$

$$2x^2 - 5x + 3 = 4x^2 + 10x + 4$$

$$0 = 2x^2 + 15x + 1$$

$$x = \frac{-15 \pm \sqrt{225 - 4(2)(1)}}{2(2)}$$

$$x = \frac{-15 \pm \sqrt{217}}{4}$$

$$\frac{x^2+6}{3} - \frac{7}{2} = \frac{x+10}{2} \qquad ① LCD\ 6.$$

$$\frac{x^2+6}{3} = \frac{x+10}{2} + \frac{7}{2}$$

$$\frac{x^2+6}{3} = \frac{x+17}{2}$$

$$2x^2 + 12 = 3x + 51$$

$$2x^2 - 3x - 39 = 0$$

$$x = \frac{3 \pm \sqrt{9 \cdot 4(2)(-39)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 + 312}}{4} = \frac{3 \pm \sqrt{321}}{4}$$

$$\frac{4}{\overline{)11}}$$
$$\frac{-12}{39}$$
$$7$$
$$39$$
$$\frac{\times 8}{312}$$

$$\frac{2x-1}{2x+1} + \frac{x+1}{x+3} = \frac{3x-1}{2x+1} + \frac{1}{6} \qquad ① LCD\ 6(x+3)(2x+1)$$

$$\frac{2x-1}{2x+1} - \frac{3x-1}{2x+1} = \frac{1}{6} - \frac{x+1}{x+3}$$

$$\frac{-x}{2x+1} = \frac{1}{6} - \frac{x+1}{x+3}$$

$$-x(6)(x+3) = (2x+1)(x+3) - (x+1)(6)(2x+1)$$

$$-6x^2 - 18x = 2x^2 + 7x + 3 - 6(2x^2 + 3x + 1)$$

$$0 = 8x^2 + 25x + 3 - 12x^2 - 18x - 6$$

$$0 = -4x^2 + 7x - 3$$

$$0 = 4x^2 - 7x + 3 \qquad 0 = (4x - 1)(x - 3)$$

$$4 \quad x \quad -3 \qquad x = \frac{1}{4}, x = 3$$
$$1 \qquad -1$$

g) $\dfrac{2x-3}{x-1} - \dfrac{x-1}{x+2} = \dfrac{2x-5}{x+2} + \dfrac{2-x}{1-x}$

$\dfrac{2x-3}{x-1} - \dfrac{2-x}{1-x} = \dfrac{2x-5}{x+2} + \dfrac{x-1}{x+2}$

$\dfrac{2x-3}{x-1} + \dfrac{2-x}{x-1} = \dfrac{3x-6}{x+2}$

$\dfrac{x-1}{x-1} = \dfrac{3x-6}{x+2}$

$1 = \dfrac{3x-6}{x+2}$

$x+2 = 3x-6$

$8 = 2x$

$\boxed{4 = x}$

h) $\dfrac{3x^2}{x^2-1} + \dfrac{2x^2}{x^2+5x+6} = \dfrac{4}{x+3}$

$\dfrac{3x^2}{(x+1)(x-1)} + \dfrac{2x^2}{(x+2)(x+3)} = \dfrac{4}{(x+3)}\dfrac{(x+2)}{(x+2)}$

$\dfrac{3x^2}{(x+1)(x-1)} = \dfrac{-2x^2+4x+8}{(x+2)(x+3)}$

$3x^2(x+2)(x+3) = (x^2-1)(-2x^2+4x+8)$

EQN LOOKS PRETTY UGLY, I'll WRITE OUT THE soln AT THE BOTTOM.

$\dfrac{x-2}{x-3} + \dfrac{x-3}{x-2} = \dfrac{2x^2}{x^2-5x+6}$

$\dfrac{x-2}{x-3} + \dfrac{x-3}{x-2} = \dfrac{2x^2}{(x-3)(x-2)}$

$(x-2)(x-2) + (x-3)(x-3) = 2x^2$

$x^2-4x+4 + x^2-6x+9 = 2x^2$

$13 = 10x$

$\boxed{\dfrac{13}{10} = x}$

$\dfrac{3x+1}{x^2-2+x} = \dfrac{2x-3}{x^2-x-6} - \dfrac{5}{x^2-4x+3}$

$\dfrac{3x+1}{(x+2)(x-1)} = \dfrac{2x-3}{(x-3)(x+2)} - \dfrac{5}{(x-3)(x-1)}$

$(3x+1)(x-3) = (2x-3)(x-1) - 5(x+2)$

$3x^2-8x-3 = 2x^2-5x+3 - 5x-10$

$x^2+2x+4 = 0$

$x = \dfrac{-2 \pm \sqrt{2-4(4)}}{2}$ ← No soln

2. Solve each of the following equations. Indicate any extraneous roots if any: For which value of "x" will $\dfrac{3+x}{4+x}$ and $\dfrac{6+x}{8+x}$ be equal?

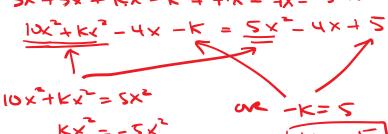$\dfrac{3+x}{4+x} = \dfrac{6+x}{8+x}$

$(3+x)(8+x) = (6+x)(4+x)$

$24+11x+x^2 = 24 + 10x + x^2$

$11x = 10x$

$\therefore \boxed{x = 0}$

3. Solve for "k":

$$\frac{3}{x-1}+\frac{k}{x}+\frac{7}{x+1}=\frac{5x^2-4x+5}{x^3-x}$$

① $x^3-x = x(x^2-1)$
$= (x)(x+1)(x-1)$

$3(x)(x+1) + k(x^2-1) + 7(x)(x-1) = 5x^2-4x+5$

$3x^2+3x + kx^2-k + 7x^2-7x = 5x^2-4x+5$

$10x^2+kx^2-4x-k = 5x^2-4x+5$

$10x^2+kx^2 = 5x^2$

$kx^2 = -5x^2$

$\boxed{k = -5}$

or $-k = 5$

$\boxed{k = -5}$

4. Jason and Thomas both work in a cleaning company. If they work separately, Jason can clean a house in 6 hours and Thomas can finish the same job in 5 hours. How much would they need if they worked together?

① JASON TAKES '6' HOURS

② THOMAS TAKES '5' HOURS

③ 'x' AMOUNT OF TIME TO COMPLETE

$$\frac{5x}{30} + \frac{6x}{30} = \frac{30}{30}$$

$11x = 30$

$x = \frac{30}{11} = 2\frac{8}{11}$ HOURS

$$\frac{x}{6} + \frac{x}{5} = 1 \leftarrow \text{100\% complete}$$

% completed by JASON   % compl. by THOMAS

5. Solve for "x": $1+\dfrac{1}{1+\dfrac{1}{x}+\dfrac{1}{2x}}=\dfrac{7}{5}$

$$\frac{1}{1+\frac{1}{x}+\frac{1}{2x}} = \frac{2}{5}$$ ① SUBTR. 1

$1+\frac{1}{x}+\frac{1}{2x} = \frac{5}{2}$ ② Flip

$\frac{1}{x}+\frac{1}{2x} = \frac{3}{2}$ ③ SUBTR. 1

$\frac{3}{2x} = \frac{3}{2}$ ④ Cross Multip.

$2x = 2$

$\boxed{x = 1}$

⑤ N.P.V. $x \neq 0$

$1+\frac{1}{x}+\frac{1}{2x} \neq 0$

$2x+2+1 \neq 0$

$2x \neq -3$

$x \neq -\frac{3}{2}$